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# Free convective mass transfer at up-pointing truncated cones

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**Abstract**

Free convective mass transfer at the individual surfaces and total surface of up-pointing truncated cones of different height and base diameter were experimentally studied using the limiting diffusion current technique. It was found that the mass transfer rate is highest for the upward-facing horizontal surface. The experimental mass transfer coefficient for the combination of downward-facing horizontal and conical surface is lower than that obtained from the equivalent summed separate surfaces due to the fact that the conical surface is exposed to solution which has already been depleted in cupric ions. The total mass transfer was successfully correlated using a single characteristic dimension (defined as a surface area divided by perimeter projected onto horizontal plane) which takes into account all the relevant dimensions of a truncated cone, i.e. the height and two diameters. 

*Keywords:* Free convection; Mass transfer; Heat transfer; Truncated cone; Electrochemical method

## 1. Introduction

Knowledge of free convective heat transfer and, therefore, the prediction of heat losses from various objects of complex geometry is very important, for instance, in design for heat dissipation from electronic components [1,2]. Mass transfer measurements are known to provide good simulation of free convective heat transfer [3]. The limiting diffusion current technique of mass transfer measurement is attractive as a cold modelling method for heat transfer since the measurements are usually simpler, cheaper and speedier than direct heat transfer measurements. Moreover, it is also easy to determine the heat transfer performance of the individual surfaces in a complex multi-surface geometry. The work of Worthington et al. [2] on the mass transfer simulation of heat transfer at cuboid shapes was prompted by an industrial need for data concerning the passive cooling of electronic components. However, such components may also be of cylindrical shape and non-uniform cross-section. In this work, therefore, we treat a shape with two horizontal surfaces and a varying diameter, the truncated cone.

This work continues previous studies devoted to free convective mass transfer at complex 3D objects such as vertical cylinders [4], up- and down-pointing pyramids [5,6] and down-pointing truncated cones [7], where the conical surface has negative inclination. In this particular study, the truncated cone is investigated in up-pointing orientation, where the inclination of the conical surface is positive.

The aim of this study was to measure mass transfer rates at the individual and total surfaces and to correlate data using an appropriate characteristic dimension.

## 2. Experimental

The experiments were performed in a rectangular 3 $dm^3$ glass container of dimensions 15 cm × 30 cm and height 25 cm. The technique used involved the cathodic deposition of cupric ions from a $CuSO_4/H_2SO_4/H_2O$ electrolyte. In order to vary the density difference between bulk fluid and surface, the cupric ion concentration was varied from 0.03 to 0.14 $mol\,dm^{-3}$. Each solution contained 1.5 $mol\,dm^{-3}$ sulphuric acid as supporting electrolyte. The electrolyte temperature was carefully measured and always lay within the range 19–22 °C being constant to ±0.1 °C during each individual experiment. The electrodes were machined from solid brass. Each electrode was supported using a 2 mm diameter brass wire glued into a hole in the centre of the horizontal base. This wire also served as a current carrier. The wire was lacquered to insulate it from the electrolyte. Two copper plates situated either side of the cones served as a counter electrode (anode). The arrangement of the apparatus is similar to that described previously

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**Nomenclature**

| | |
|---|---|
| $A$ | surface area ($cm^2$) |
| $c_b$ | bulk concentration of cupric ions ($mol\,dm^{-3}$) |
| $d_2$ | diameter of the top surface of truncated cone (cm) (Fig. 2) |
| $d_1$ | diameter of the bottom surface of truncated cone (cm) (Fig. 2) |
| $D$ | diffusion coefficient of cupric ions ($m^2\,s^{-1}$) |
| $F$ | Faraday constant (96487 °C $mol^{-1}$) |
| $g$ | gravitational acceleration ($m\,s^{-2}$) |
| $Gr_{L_w}$ | Grashof number based on characteristic length $L_w$, $Gr_{L_w} = \Delta\rho g L_w^3 \rho/\mu^2$ |
| $H$ | height of truncated cone (cm) (Fig. 2) |
| $I_L$ | limiting diffusion current ($mA\,cm^{-2}$) |
| $k$ | mass transfer coefficient ($m\,s^{-1}$) |
| $K$ | thermal conductivity ($W\,m^{-1}\,K^{-1}$) |
| $L$ | length of conical surface of truncated cone (cm) |
| $L_w$ | characteristic length defined by Weber et al. [10], $L_w$: surface area/perimeter projected onto the horizontal plane (cm) |
| $n$ | charge number of cupric ion, $n = 2$ |
| $Nu$ | Nusselt number, $Nu = hL/K$ |
| $p$ | pressure (Pa) |
| $Ra$ | Rayleigh number, $Ra = Gr \times Sc$ |
| $Sc$ | Schmidt number, $\mu/\rho D$ |
| $Sh_{L_w}$ | Sherwood number, $kL_w/D$ |
| $T$ | electrolyte temperature (K) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | inclination angle from the vertical (°) (Fig. 2) |
| $\mu$ | dynamic viscosity ($kg\,m^{-1}\,s^{-1}$) |
| $\nu$ | kinematic viscosity ($m^2\,s^{-1}$) |
| $\rho$ | density ($kg\,m^{-3}$) |
| $\Delta\rho$ | density difference between bulk solution and interface ($kg\,m^{-3}$) |

*Subscripts*

| | |
|---|---|
| C | conical surface of truncated cone |
| D | down-facing horizontal base of truncated cone |
| DC | combination of down-facing horizontal and conical surface |
| T | total surface of truncated cone |
| U | up-facing horizontal base of truncated cone |

[5–7] and is shown in Fig. 1. The actual $Cu^{2+}$ concentration was periodically determined by spectrophotometric analysis.

The usual electrical circuit for limiting current measurement was employed, consisting of a d.c. power supply with a voltage regulator, a high impedance voltmeter and a multi-range ammeter. Limiting currents were obtained by the well-known procedure, which has been reported in detail previously [8]. The anode acted as a reference electrode in view of its high area compared to that of the cathode. Under such conditions, polarisation is negligible at the anode and the cell current–voltage relationship depends only on the conditions prevailing at the cathode. The onset of the limiting current was sharp and reproducible. The geometrical parameters of the truncated cones are illustrated in Fig. 2. Table 1 lists the dimensions used. In this investigation, $H$ ranged from 0.2 to 8.0 cm and $\alpha$ from 11° to 83°. Inclination angles are counted positive to indicate the up-facing orientation of the conical surface and for consistency with the work of Patrick et al. [9]. The mass transfer controlled limiting currents at the conical surface and at the top and bottom horizontal surfaces were measured separately and in combination. Surfaces not required to be active were stopped off with lacquer (Lacomit).

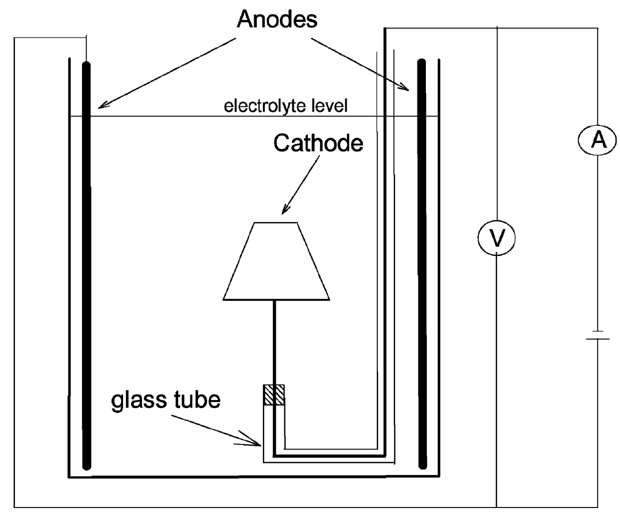


Fig. 1. Apparatus.

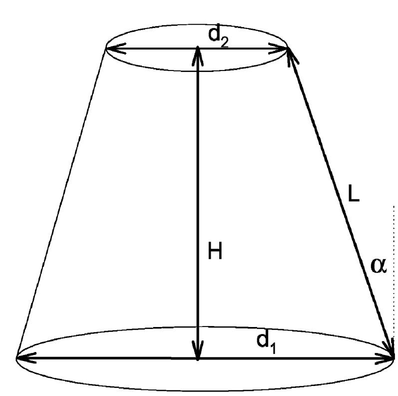


Fig. 2. Geometry of up-pointing truncated cone.

Table 1
Geometric parameters of truncated cones

| Truncated cone | $d_1$ (cm) | $d_2$ (cm) | $H$ (cm) | $L$ (cm) | $\alpha$ (°) | $A_C$ (cm$^2$) | $A_D$ (cm$^2$) | $A_U$ (cm$^2$) | $A_T$ (cm$^2$) | $d_1/d_2$ | $L_w$ (cm) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 3.0 | 1.0 | 0.2 | 1.02 | 78.7 | 6.4 | 7.1 | 0.8 | 14.3 | 3.0 | 1.51 |
| 2 | 3.0 | 0.9 | 0.5 | 1.12 | 63.4 | 7.0 | 6.6 | 0.7 | 14.3 | 3.2 | 1.52 |
| 3 | 2.9 | 1.0 | 1.0 | 1.42 | 45.0 | 8.9 | 7.1 | 0.8 | 16.7 | 3.0 | 1.78 |
| 4 | 3.0 | 1.0 | 2.0 | 2.25 | 27.0 | 14.1 | 7.1 | 0.8 | 22.0 | 3.0 | 2.41 |
| 5 | 3.0 | 1.0 | 4.0 | 4.12 | 14.0 | 25.9 | 7.1 | 0.8 | 33.8 | 3.0 | 2.35 |
| 6 | 3.0 | 1.0 | 8.0 | 8.06 | 7.0 | 50.6 | 7.1 | 0.8 | 58.5 | 3.0 | 6.21 |
| 7 | 3.0 | 2.0 | 0.5 | 0.71 | 45.0 | 5.6 | 7.1 | 3.1 | 15.8 | 1.5 | 1.67 |
| 8 | 3.0 | 0.5 | 1.25 | 1.77 | 45.0 | 9.7 | 7.1 | 0.2 | 16.7 | 6.0 | 1.77 |
| 9 | 5.0 | 0.5 | 2.25 | 3.18 | 45.0 | 27.5 | 19.6 | 0.2 | 47.3 | 10.0 | 3.01 |

## 3. Results and discussion

### 3.1. Mass transfer measurement

For each experiment, the mass transfer coefficient was calculated from the measured limiting current using the equation

$$k = \frac{I_L}{AnFc_b} \tag{1}$$

The area in this equation was the total available for mass transfer for the particular experiment. The correction for the attachment of the supporting wire was small, never being more than 2% of the exposed area, and usually being much less.

The effect of the truncated cone height $H$ on the mass transfer coefficient for the separate surfaces of up-pointing truncated cones of constant top (1 cm) and bottom base diameter (3 cm), cones 1–6, is shown in Fig. 3. From the geometry of a truncated cone, it is clear that with increasing height (from 0.2 to 8 cm), the inclination angle decreases (from 78.7° to 7°). The superior mass transfer performance of the up-facing horizontal surface is immediately apparent, especially compared with the down-facing horizontal surface. Due to the constant horizontal diameters, mass transfer coefficients for these surfaces do not depend on cone height.

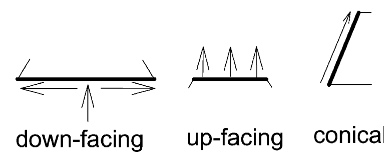


Fig. 4. Schematic flow around individual surfaces: (a) down-facing surface; (b) up-facing surface; (c) conical surface.

A down-facing horizontal surface hinders the upward electrolyte flow which has to flow radially outwards (Fig. 4a) and this is responsible for the low mass transfer coefficient. For up-facing horizontal surfaces, turbulent flow behaviour is dominant (Fig. 4b) and this is represented by high mass transfer coefficients. The mass transfer coefficient for the conical surface decreases with increasing length which is typical for laminar flow (Fig. 5). Both increase in height

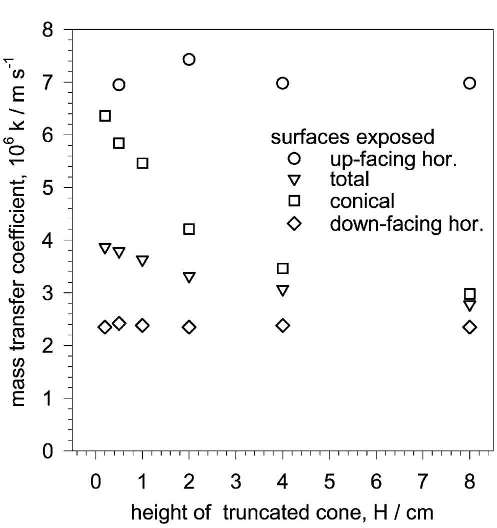


Fig. 3. Dependence of mass transfer coefficient on truncated cone height for individual and total surfaces (cupric ion concentration = 0.14 mol dm$^{-3}$).

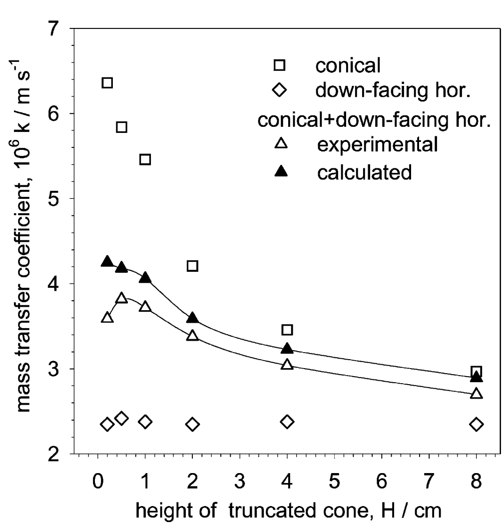


Fig. 5. Dependence of mass transfer coefficient on truncated cone height for the conical and down-facing horizontal surfaces separately and in combination (cupric ion concentration = 0.14 mol dm$^{-3}$).

and decrease in inclination angle lead to a decrease in mass transfer coefficient. The mass transfer coefficient for the total surface decreases with increasing conical height and is significantly reduced by the presence of the down-facing horizontal base.

The effect of the truncated cone height $H$ on the mass transfer coefficient for the conical and down-facing horizontal surfaces and their combination is shown in Fig. 5. It is apparent that the mass transfer rate for the combination is strongly affected by the down-facing surface. The experimental mass transfer coefficient ($k_{\mathrm{DC,\,exp}}$) for a combination of both surfaces was compared with that calculated assuming that the down-facing horizontal and the conical surface do not influence each other, i.e. both surfaces contribute to the overall mass transfer proportionally to their surface area

$$k_{\mathrm{DC,\,calc.}} = \frac{A_{\mathrm{D}}k_{\mathrm{D}} + A_{\mathrm{C}}k_{\mathrm{C}}}{A_{\mathrm{DC}}} \tag{2}$$

From Fig. 5, it can be seen that experimental mass transfer coefficients are about 10–15% lower than those calculated and the difference is particularly high for truncated cone height 0.2 cm, where the ratio of bottom and conical surface is the highest. The reason is that the flow of the fluid around the down-facing horizontal surface means that the conical surface is exposed to solution which has already been depleted of cupric ions (Fig. 6). This boundary layer spillover effect causes the mass transfer for a combination to be lower than that for the equivalent summed separate surfaces.

### 3.2. Overall mass transfer correlation

Different characteristic lengths for calculation of $Sh$ and $Ra$ number were considered. The use of truncated cone height does not take into account the fact that the horizontal surfaces are also active. On the other hand, using the diameter of a horizontal surface does not take into account the effect of the conical surface. Furthermore, there are two horizontal surfaces each of different diameter. Therefore, the total mass transfer was correlated using the single characteristic dimension previously defined by Weber et al. [10] which takes into account all the relevant dimensions of a truncated cone (height and two diameters) in the form given in the following equation:

$$L_{\mathrm{w}} = \frac{\text{surface area}}{\text{perimeter projected onto horizontal plane}} = \frac{A_{\mathrm{D}} + A_{\mathrm{C}} + A_{\mathrm{U}}}{\pi d_1} \tag{3}$$

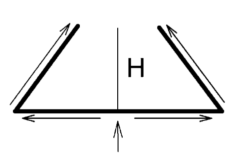


Fig. 6. Schematic flow around the combination of down-facing and conical surface.

Table 2
Uncertainty of measured quantities

| Quantity | Nominal value | Uncertainty | Error (%) |
|---|---|---|---|
| $d$ | 1.0 cm | 0.01 cm | 1 |
| $H$ | 0.5 cm | 0.01 cm | 2 |
| $H$ | 8.0 cm | 0.01 cm | 0.13 |
| $c_{\mathrm{b}}$ | 0.16 mol dm$^{-3}$ | 0.0005 mol dm$^{-3}$ | 0.3 |
| $c_{\mathrm{b}}$ | 0.03 mol dm$^{-3}$ | 0.0005 mol dm$^{-3}$ | 2.0 |
| $I_{\mathrm{L}}$ | 20 mA cm$^{-2}$ | 0.3 mA cm$^{-2}$ | 1.5 |

A Rayleigh number and a Sherwood number based on this characteristic length $L_{\mathrm{w}}$ can then be defined as

$$Ra_{L_{\mathrm{w}}} = Gr_{L_{\mathrm{w}}}Sc = \frac{g\Delta\rho L_{\mathrm{w}}^3\rho}{\mu^2}\frac{\mu}{\rho D} \tag{4}$$

$$Sh_{L_{\mathrm{w}}} = \frac{kL_{\mathrm{w}}}{D} \tag{5}$$

The diffusivity of the $Cu^{2+}$ ions was calculated using the data of Wilke et al. [11]. Electrolyte density and viscosity were calculated using data of Eisenberg et al. [12]. The $\Delta\rho$ terms were taken from Wilke et al. [11]. The effect of migration on the copper deposition rate was negligible [13].

The uncertainties in the measured quantities are expressed in Table 2. In addition, the uncertainty in the fluid properties $D$, $\mu$ and $\Delta\rho$ was taken as 0.5%. The uncertainty in the determination of the Sherwood number and Rayleigh number depends on the concentration of cupric sulphate solution. For 0.02 mol dm$^{-3}$ it was estimated as 4%; for 0.16 mol dm$^{-3}$ as 1.5%.

The mass transfer data for entire truncated cones from 1 to 9 are plotted in terms of Sherwood number vs. Rayleigh number in Fig. 7 using the characteristic length, $L_{\mathrm{w}}$. Least square analysis gives the correlation

$$Sh_{L_{\mathrm{w}}} = 0.43Ra_{L_{\mathrm{w}}}^{0.265} \tag{6}$$

for $Ra_{L_{\mathrm{w}}}$ in the range $1.8 \times 10^8$–$5.9 \times 10^{10}$.

Weber et al. [10] measured mass transfer coefficients for a variety of spherical and non-spherical 3D objects (spheres,

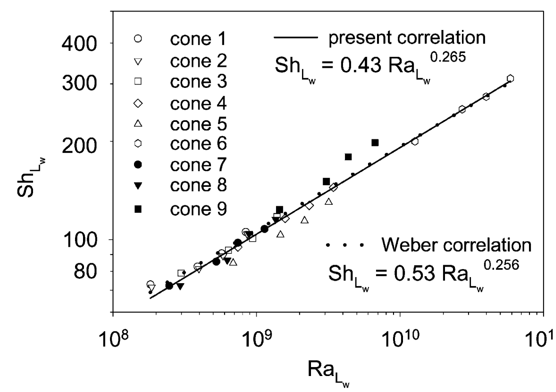


Fig. 7. Mass transfer correlation for up-pointing truncated cone using the Weber characteristic dimension.

cylinders and cones). Using the characteristic length $L_w$ in calculating *Sh* and *Gr*, they obtained the correlation

$$Sh_{L_w} = 0.53 Ra_{L_w}^{0.256} \tag{7}$$

The exponent in the present correlation (Eq. (6)) is about 4% higher and the constant about 20% lower than for the Weber correlating equation. Fig. 7 illustrates agreement between the two correlating equations.

The exponent in Eq. (6) is close to 0.265 showing that flow at the conical surface remains laminar and attached. Nevertheless it can be seen that points for truncated cone 9 are a long away off the correlation. Patrick and Wragg [14] measured free convection mass transfer at up-facing conical surfaces. They identified critical Rayleigh numbers for the transition of attached laminar free convection boundary layer at the sloping surfaces to give a pseudo-turbulent multi-plumed flow. Critical *Ra* number was found to decrease with increasing inclination. *Ra* numbers for truncated cone 9 ($2–7 \times 10^9$) are significantly higher (about 1 order) than the critical *Ra* number corresponding to inclination 45° ($1–3 \times 10^8$ [14]). This means that for this truncated cone, transition from attached laminar to turbulent flow occurs and this inevitably leads to a notable increase in mass transfer rate.

Heat transfer and mass transfer can be expressed via the analogy equation

$$Nu = f(Gr, Pr) = f(Gr, Sc) = Sh \tag{8}$$

where Nusselt number *Nu* represents the dimensionless heat transfer coefficient *h*. Thus heat transfer from passively cooling objects such as electronic components can be calculated using the heat transfer analogy form of the present correlation (Eq. (6)). However, the mass transfer results obtained using the electrochemical technique correspond to high Schmidt numbers (≅2000) and must be applied with some caution. One advantage of the electrochemical analogy method is that for heat transfer prediction with air ($Pr \cong 0.7$), thermal modelling of large objects on a much smaller geometric scale is facilitated.

## 4. Conclusions

1. The mass transfer coefficient at truncated cones is higher for the up-facing horizontal than for the down-facing horizontal surface.
2. The measured mass transfer coefficient for a combination of conical and down-facing horizontal surfaces is lower than that obtained from calculation based on the separate surfaces due to the fact that the conical surface is exposed to solution which has already been depleted of cupric ions.
3. Overall mass transfer data for up-pointing truncated cones obtained using a single characteristic length are in good agreement with the Weber general correlation [10] for 3D objects.

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